$$\int \frac{5e^{3x}+13e^{2x}+11e^{x}-2}{(e^{x}+1)(e^{2x}+2e^{x}+2)}\,dx = \int \frac{5y^3+13y^2+11y-2}{(y+1)\,y\,(y^2+2y+2)}\,dy =$$ Rozklad na parc. zlomky

$e^x = y$
$e^x dx = dy$
Substituce

$$\int \frac{A}{y+1} + \frac{B}{y} + \frac{Cy+D}{y^2+2y+2}\,dy = 5\log|y+1| - \log|y| + I \stackrel{c}{=} *$$

$$Ay(y^2+2y+2) + B(y+1)(y^2+2y+2) + (Cy+D)(y+1)y = 5y^3+13y^2+11y-2$$

$y=0: 2B=-2 \Rightarrow \underline{B=-1}$
$y=-1: -A=-5 \Rightarrow \underline{A=5}$
} Dosazování kořenů jmenovatele. Je vhodná metoda pokud kořen je reálný nebo ryze imaginární.

$y=1: 5.1.5 + (-1).2.5 + 2C+2D = 2C+2D+15 = 27 \Rightarrow C+D=6$
$y=-2: 5.(-2).2 + (-1).(-1).2 + (-2C+D).(-1)(-2) = -18-4C+2D = -12 \Rightarrow -2C+D = 3$
$\Rightarrow 3D=15 \Rightarrow \underline{D=5} \Rightarrow \underline{C=1}$

$$I = \int \frac{Cy+D}{y^2+2y+2}\,dy = \frac{1}{2}\log|y^2+2y+2| + 4\int \frac{1}{(y+1)^2+1}\,dy \stackrel{c}{=}$$
$$\frac{1}{2}\log|y^2+2y+2| + 4\,\mathrm{arctg}(y+1).$$

$$\boxed{\int \frac{1}{2}\cdot\frac{2y+2}{y^2+2y+2}\,dy + 4\int\frac{1}{y^2+2y+2}\,dy}$$

$$* = 5\log(e^{x}+1) - x + \frac{1}{2}\log(e^{2x}+2e^{x}+2) + 4\,\mathrm{arctg}(e^{x}+1),\ x\in\mathbb{R}.$$